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gallons of ink have been wasted, i. e., the production of pandiagonal and regular squares of the sixth order. It is impossible to distribute six marks among the thirty-six cells of this square so that one and only one mark shall appear in every column, row and diagonal. Hence a *primitive* pandiagonal magic square of this order is excluded by a geometrical necessity. In this case the natural series of numbers is not adapted to construct pandiagonal squares of this order. That the difficulty is simply an arithmetical one is proven by the fact that 6×6 pandiagonal squares can be made with *other series*, as shown in Fig. 16. We are indebted to Dr. C. Planck for this interesting square which is magic in its six rows, six columns and twelve diagonals, and is also four-ply and nine-ply, i. e., any square group of four or nine cells respectively, sums four or nine times the mean. It is constructed from a series made by arranging the numbers 1 to 49 in a square and eliminating all numbers in the central line and column, thus leaving thirty-six numbers as follows:

1	2	3	5	6	7
8	9	10	12	13	14
15	16	17	19	20	21
29	30	31	33	34	35
36	37	38	40	41	42
43	44	45	47	48	49

Fig. 17 shows the completed square which is illustrated in skeleton form in Fig. 1. All the sub-squares are faultless except the small internal 3×3 , in which one diagonal is incorrect.

FRIERSON, LA.

L. S. FRIERSON.

TWO MORE FORMS OF MAGIC SQUARES.

SERRATED MAGIC SQUARES.

The curious form of magic squares, which is to be described here, is a style possessing a striking difference from the general form of magic squares.

To conform with the saw-tooth edges of this class of squares, I have ventured to call them "serrated" magic squares.

A square containing the series 1, 2, 3, 4, . . . 41 is shown in Fig. 1. Its diagonals are the horizontal and vertical series of nine numbers, as A in Fig. 2. Its rows and columns are zigzag as

shown at B, and are sixteen in number, a quantity which is always equal to the number of cells which form the serrations.

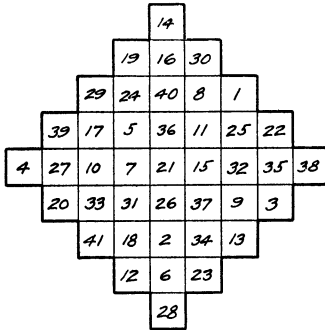


Fig. 1.

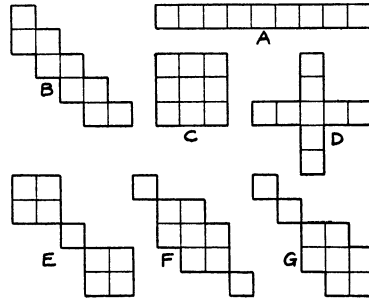


Fig. 2.

All of this class of squares must necessarily contain the two above features.

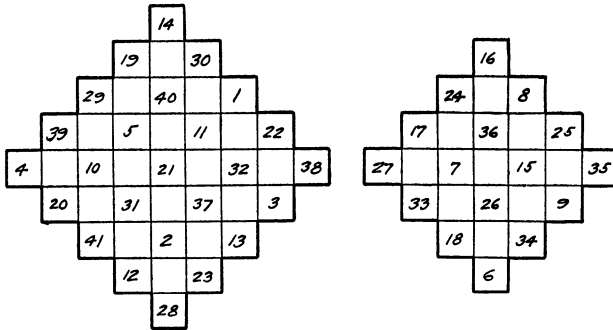


Fig. 3.

But, owing to its Nasical formation, Fig. 1 possesses other features as follows:

1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18
19	20	21	22	23	24	25	26	27
28	29	30	31	32	33	34	35	36
37	38	39	40	41				

Fig. 4.

There are nine summations each of the square and cruciform, as at C and D in Fig. 2, the centers of which are 40, 11, 32, 5, 21,

37, 10, 31 and 2 respectively. Of E and F there are six summations each, and of the form G there are twelve summations.

This square was formed by the interconcentric position of the two Nasik squares shown in Fig. 3, and the method of selecting their numbers is clearly shown in Fig. 4.

There are numerous other selections for the sub-squares and the summations are not necessarily constant. This is shown by the following equations.

Let N and n equal the number of cells on a side of the large and small squares respectively, and let Σ equal the summations.

Then, when the means of each sub-square are equal

$$\Sigma = \frac{(1+N^2+n^2)(N+n)}{2}$$

When the large square has the first of the series and the small square has the last of the series

$$\Sigma = \frac{N(1+N^2)}{2} + \frac{n(1+n^2)}{2} + N^2n$$

When the large square has the last of the series and the small square has the first of the series

$$\Sigma = \frac{N(1+N^2)}{2} + \frac{n(1+n^2)}{2} + Nn^2$$

Only in such squares that fit the first equation, is it possible to

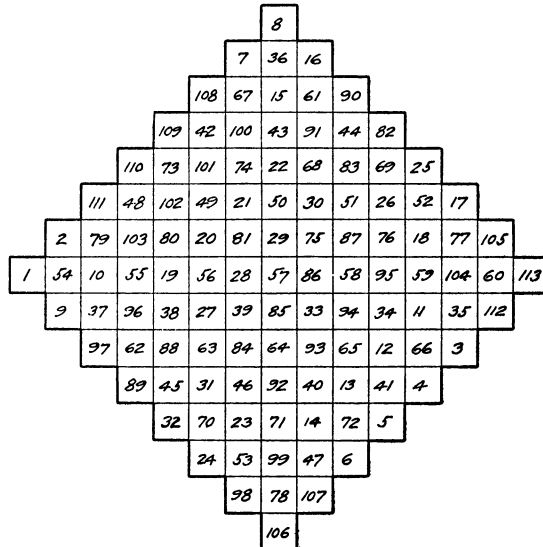


Fig. 5.

have complementary pairs balanced about the center; in other words known as regular or associated squares.

Fig. 5 is one of this class and has summations of 855. In this case the mean of the series was used in the 7×7 sub-square and the remaining extremes made up the 8×8 square.

			4			
		23	14	15		
	24	19	10	5	8	
1	6	9	13	17	20	25
	18	21	16	7	2	
		11	12	3		
			22			

Fig. 6.

			4			
		14	18	9		
	15	25	7	23	5	
1	20	6	21	11	22	16
	12	19	10	17	2	
		8	24	3		
			13			

Fig. 7.

			13			
		23	2	18		
	24	9	16	7	14	
10	4	15	5	20	6	25
	21	3	19	1	11	
		17	8	12		
			22			

Fig. 8.

Figs. 6, 7, and 8 are the smallest possible examples of serrated squares. Fig. 6 is regular and is formed with the first of the above mentioned equations, and its summations are 91. Fig. 7 is formed with the second equation and its summations are 97. Fig. 8 is formed with the third equation and its summations are 85.

MAGIC SQUARES WITH THE ODD NUMBERS IN SEQUENTIAL SERIES.

During the last year the writer has noticed in a weekly periodical, a few examples of magic squares in which all of the odd numbers are arranged sequentially in the form of a square, the points of which meet the centers of the sides of the main square and the even numbers filling in the corners as shown in Fig. 3.

5	6	7	1	2	3	4
6	7	1	2	3	4	5
7	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	1
3	4	5	6	7	1	2
4	5	6	7	1	2	3

Fig. 1.

3	2	1	0	6	5	4
4	3	2	1	0	6	5
5	4	3	2	1	0	6
6	5	4	3	2	1	0
0	6	5	4	3	2	1
1	0	6	5	4	3	2
2	1	0	6	5	4	3

Fig. 2.

26	20	14	1	44	38	32
34	28	15	9	3	46	40
42	29	23	17	11	5	48
43	37	31	25	19	13	7
2	45	39	33	27	21	8
10	4	47	41	35	22	16
18	12	6	49	36	30	24

Fig. 3.

These articles merely showed the completed square and did not show or describe any method of construction.

A few simple methods of constructing these squares are described below, which may be found of some interest.

To construct such squares, n must necessarily be odd, as 3, 5, 7, 9, 11 etc.

A La Hireian method is shown in Figs. 1, 2, and 3, in which the first two figures are primary squares used to form the main square, Fig. 3. We begin by filling in the cells of Fig. 1, placing 1 in the top central cell and numbering downward 1, 2, 3 to 7 or n . We now repeat these numbers pan-diagonally down to the left, filling the square.

Fig. 2 is filled in the same manner, only that we use the series 0, 1, 2, to 6 or $n-1$ in our central vertical column, and repeat these pan-diagonally down to the right. The cell numbers in Fig. 2 are then multiplied by 7 or n and added to the same respective cell numbers of Fig. 1, which gives us the final square Fig. 3.

				18		
			22		24	
		1	2	3	4	5
	10	6	7	8	9	10
14		11	12	13	14	15
	20	16	17	18	19	20
		21	22	23	24	25
			2		4	
				8		

Fig. 4.

Another method is shown in Fig. 4 where we have five sub-squares placed in the form of a cross. The central one of these is filled consecutively from 1 to n^2 . We then take the even numbers of the upper quarter, in this case 2, 8 and 4, and place them in the same respective cells in the lower sub-square. The lower quarter or 22, 18 and 24, are placed in the upper square. Likewise the left-hand quarter is placed in the right-hand square, and the right-hand quarter in the left-hand square. This gives us the required square, which is shown in heavy numbers.

A third method is to write the numbers consecutively, in the form of a square, over an area of adjacent squares as in Fig. 5. The mean of the series must be placed in the center cell of the central or main square and the four next nearest to the center must find their places in the corner cells of the main square, which consequently governs the spacing in writing the series. We then re-

move all these numbers to the same respective cells in the main square, and this gives us the square shown in Fig. 6.

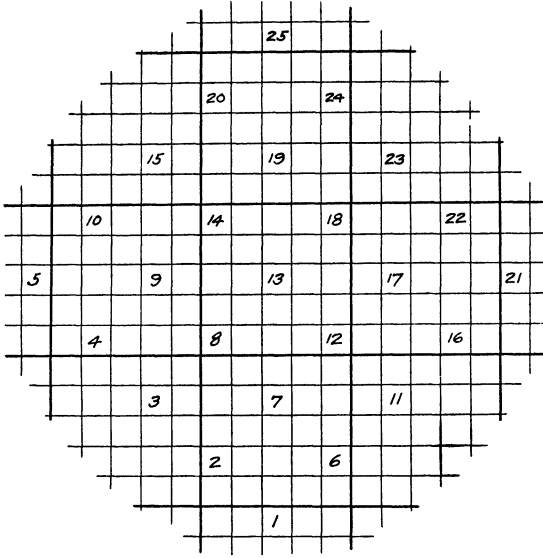


Fig. 5.

14	10	1	22	18
20	11	7	3	24
21	17	13	9	5
2	23	19	15	6
8	4	25	16	12

Fig. 6.

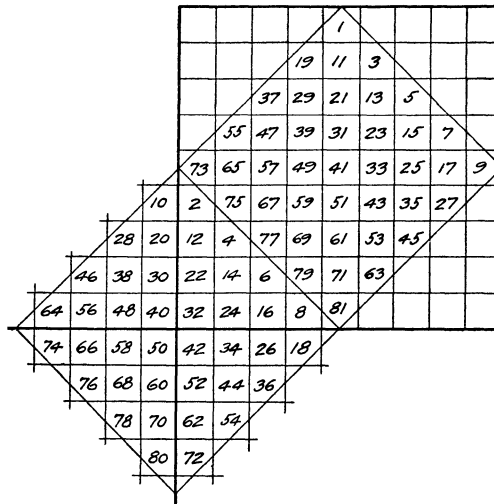


Fig. 7.

This last method is not preferable, owing to the largeness of the primary arrangement, which becomes very large in larger squares.

It might however be used in the break-move style,¹ where the steps are equal to the distance from the center cell to the corner cell, and the breakmoves are one cell down when 1 is at the top.

What seems to be the most simple method is shown in Fig. 7, where the odd numbers are written consecutively in the main square and directly following in the same order of progression, the even numbers are written.

The even numbers necessarily run over into three adjacent sub-

42	34	26	18	1	74	66	58	50
52	44	36	19	11	3	76	68	60
62	54	37	29	21	13	5	78	70
72	55	47	39	31	23	15	7	80
73	65	57	49	41	33	25	17	9
2	75	67	59	51	43	35	27	10
12	4	77	69	61	53	45	28	20
22	14	6	79	71	63	46	38	30
32	24	16	8	81	64	56	48	40

Fig. 8.

squares. These are removed to the same respective cells in the main square, the result of which is shown in Fig. 8.

It will be noticed that all these methods give identically the same results, which I believe are the only possible forms of this style of squares.

The summations of Fig. 3 are 175, the summations of Figs. 4 and 6 are 65, and the summations for Fig. 8 are 369. Also, all complementary pairs are balanced about the center.

SCHENECTADY, N. Y.

HARRY A. SAYLES.

WORK TO BE DONE IN BUDDHIST CRITICISM.

AN APPEAL TO CHINESE SCHOLARS.

Perhaps there is nothing more romantic in the history of religion than the spectacle of a Parthian prince renouncing his throne in A. D. 149 and going to China as a Buddhist monk. He spent his life in his adopted country, translating parts of the sacred writings into Chinese. According to his own *Catalogue of the Chinese Tripitaka* (Oxford, 1883), Nanjio translated 176 original works, of which

¹This style is thoroughly explained in *Magic Squares and Cubes* by Mr. W. S. Andrews.